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14. ABSTRACT
This paper is an investigation of the statistical characteristics of the amplitude of signals scattered from a randomly rough surface. When the area of illumination and the grazing angles are large the amplitude of scattered signals are found to obey Rayleigh statistics. On the other hand, when the grazing angles are very small and the resolution size is small it has been found Lognormal statistics form a good fit. However, in the intermediate domain we find that no particular distribution has overall best fit. The conclusion is based on a statistical analysis of the simulated data generated by rigorous rough surface scattering analysis along with Monte Carlo simulation of sample surfaces. The scattering calculations are based on integral equation formulation of scattering and are solved using method of moments.

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Statistical Characteristics of Microwave Signals Scattered from a Randomly Rough Surface

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Introduction

Scattering of microwave signals from rough surfaces has been extensively studied [1-3]. However, a vast majority of such studies is on the characteristics of the normalized scattering cross section. This paper is an investigation of the statistical characteristics of scattered signals. When the area of illumination is large it is seen that the scattering from a rough surface may be considered to be contributions from a large number of individual cells. One may hence appeal to the central limit theorem and show that the amplitude of the scattered signal satisfies the Rayleigh distribution. This observation was first made by Rayleigh a hundred years ago for the general case of randomly scattered signals. Beckmann demonstrated that similar arguments and deductions may be employed for the case of scattering from rough surfaces. It is apparent that certain conditions are necessary to deduce that the amplitude statistics are Rayleigh distributed. When such conditions do not exist the amplitude statistics are not Rayleigh and in such situations we would like to know which statistical distribution is most representative. It has been found that the K-distribution [4] is a good model for randomly scattered signals. We consider this and some other appropriate non-Rayleigh statistical distributions. For our study we use numerically simulated signals scattered from a randomly rough surface. We employ the moment method to estimate the parameters of the various distributions. We next use Kolmogorov-Smirnov statistic to determine which of the distributions most closely fit the simulated data.

Description of the Problem

We have a perfectly conducting randomly rough surface given as $z = \zeta(x)$ which is planar on the average. ζ is a zero-mean stationary random process independent of y. For TE case the incident field is taken as $\mathbf{E}_i = \hat{y}E_i$, where

$$E_i = \exp\{ik(x\sin\theta_i - z\cos\theta_i)[1+u(\mathbf{r})]\}\exp\left\{-\frac{1}{w^2}(x+z\tan\theta_i)\right\}, \tag{1}$$

w denotes the width of the Gaussian beam, and

$$u(\mathbf{r}) = \frac{1}{(kw\cos\theta_i)^2}\left\{\frac{2}{w^2}(x+z\tan\theta_i)^2 - 1\right\}. \tag{2}$$

Equation (1) describes a Gaussian beam often used in numerical simulations. For the TM case we have $\mathbf{H}_i = \hat{y}H_i$ where H_i is identical to the E_i in (1). Our main interest is in studying the statistical characteristics of the fields scattered by the rough surface.

Formulation

The reduced wave equations for time-harmonic waves are

$$\nabla^2\mathbf{E} + k^2\mathbf{E} = 0 \qquad \nabla^2\mathbf{H} + k^2\mathbf{H} = 0, \tag{3}$$

where k is the free space wave number. Since the surface is independent of y the polarization of the incident waves is preserved on scattering. Therefore $\mathbf{E} = \hat{y}E$ and $\mathbf{H} = \hat{y}H$. Thus we have the following scalar Helmholtz equations for TE and TM cases respectively,

$$\nabla^2 E + k^2 E = 0 \qquad \nabla^2 H + k^2 H = 0. \tag{4}$$

The boundary condition on the PEC surface is $\hat{z}\times\mathbf{E} = 0$ on the surface S. This implies that

$$E(\mathbf{r}_s) = 0 \qquad \partial_n H(\mathbf{r}_s) = 0 \qquad \mathbf{r}_s \in S, \tag{5}$$

for TE and TM cases, where $\partial_n H$ is the normal derivative on the surface S. Employing the Green's theorem we convert the differential equation system into the following integral equation system:

$$E(\mathbf{r}) = E_i(\mathbf{r}) - \int_S d\mathbf{r}' G(\mathbf{r}', \mathbf{r})\partial_{n'} E(\mathbf{r}') \qquad H(\mathbf{r}) = H_i(\mathbf{r}) + \int_S d\mathbf{r}' H(\mathbf{r}')\partial_{n'} G(\mathbf{r}', \mathbf{r}). \tag{6}$$

The essential task is to evaluate the surface fields $\partial_n E$ and H which are governed by the following integral equations:

$$\partial_n E(\mathbf{r}_s) = 2\partial_n E_i(\mathbf{r}_s) - 2 ⨍_S d\mathbf{r}' \partial_n G(\mathbf{r}', \mathbf{r}_s)\partial_{n'} E(\mathbf{r}'), \tag{7}$$

$$H(\mathbf{r}_s) = 2H_i(\mathbf{r}_s) + ⨍_S d\mathbf{r}' H(\mathbf{r}')\partial_{n'} G(\mathbf{r}', \mathbf{r}_s). \tag{8}$$

The bar across the integral sign denotes that the integral is a principal value integral. We employ the forward-backward algorithm [5,6] to obtain numerical solutions to the above integral equations. Inserting these solutions in (6) we calculate the bistatic scattering coefficients. Proceeding thus we carried out numerical simulations based on Pierson-Moskovitz surface spectrum for various surface conditions, beam widths, and angles of incidence and observation.

Statistical Characteristics of Scattered Waves

Simulated results thus obtained are valuable for understanding the nature and characteristics of various asymptotic approximations. Further, the procedure provides comprehensive statistics of scattered signals. One important quantity of interest is the probability density function (pdf) of the amplitude of scattered signals. The pdf of signals scattered from a random collection of scatterers has been investigated by Rayleigh. For the case of a randomly rough surface this problem was studied by Beckmann. The idea is to formulate the problem as a random walk process and assume that the surface is composed of contributions from a large number of independent individual cells. From this one can deduce that the distribution of amplitudes of scattered signals is Rayleigh distributed. However, this derivation is based on several assumptions and when they are violated we depart from Rayleigh statistics. There are several non-Rayleigh statistical distributions that have been postulated for such situations. Some of these are lognormal, Weibull, K-distribution, and modified K-distributions. The modified K-distributions are extensions of the K-model, obtained by using different functions to describe the texture. For our study we have considered the gamma texture, log-normal texture and the inverse gamma texture. The pdfs of the statistical models cited above are:

Log-normal:

$$f(s) = \frac{1}{s\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2} \ln^2\left(\frac{s}{\delta}\right)\right\} \qquad \text{for } s > 0$$

Weibull:

$$f(s) = \frac{c s^{c-1}}{b^c} \exp\left\{-\left(\frac{s}{b}\right)^c\right\} \qquad \text{for } s > 0$$

K-distribution:

$$f(s) = \frac{\sqrt{4\nu/\mu}}{2^{\nu-1}\Gamma(\nu)} \left\{\sqrt{\frac{4\nu}{\mu}} s\right\}^{\nu} K_{\nu-1}\left\{\sqrt{\frac{4\nu}{\mu}} s\right\} \qquad \text{for } s > 0$$

Generalized K - gamma texture:

$$f(s) = \frac{2br}{\Gamma(\nu)} \left(\frac{\nu}{\mu}\right)^{\nu b} \int_0^\infty \tau^{\nu b-2} \exp\left\{\frac{s^2}{\tau} - \left(\frac{\nu}{\mu}\tau\right)^b\right\} d\tau$$

Generalized K - Log-normal texture:

$$f(s) = \frac{s}{\sqrt{2\pi\sigma^2}} \int_0^\infty \frac{2}{\tau^2} \exp\left\{\frac{s^2}{\tau} - \frac{1}{2\sigma^2} \ln^2\left(\frac{\tau}{2m}\right)\right\} d\tau$$

Generalized K - Inverse gamma texture:

$$f(s) = \frac{2s\beta\Gamma(\alpha+1)}{(\beta s^2+1)^{(\alpha+1)}\Gamma(\alpha)}$$

The Rayleigh distribution has a one parameter pdf. All the rest have two parameter pdfs. We use the first two moments to determine these parameters from the simulated data using Pearson moments method. Let θ_1 and θ_2 be the two parameters and let μ_1 and μ_2 be the first two moments for a particular model. If s_n is the amplitude of the simulated scattered signal corresponding to the n-th realization, then the moments may be calculated as

$$m_k = \frac{1}{N_r} \sum_{n=1}^{N_r} {s_n}^k$$

where N_r is the number of realizations considered for the study. Then

$$\mu_k(\theta_1, \theta_2) = m_k \qquad k = 1, 2$$

is the pair of equations to be solved to determine the parameters.

This is a fairly simple and straightforward procedure. However, there is no information as to whether the parameters thus obtained will be unbiased and efficient. Fisher's maximum likelihood estimation method guarantees efficiency although it is not as simple to implement as the moments method. Let the N_r variate density function be f_{N_r}. Then the likelihood function is $f_{N_r}(s_1, s_2, \cdots, s_{N_r}|\theta_1, \theta_2)$. The parameters θ_1 and θ_2 are determined by solving $\partial_{\theta_1} f_{N_r} = 0$ and $\partial_{\theta_2} f_{N_r} = 0$. The next obvious question is which pdf most closely fits our simulated data. To compare the data with a particular model it is more convenient to use the cumulative distribution function (cdf). Denoting the proposed cdf as $F(s)$, and that composed by using the computed data as $F_c(s)$, we calculate the Kolmogorov-Smirnov statistic $D \equiv \sup|F_c(s) - F(s)|$. To gauge the pdf under consideration we calculate the probability that D is larger than the observed value, which is given as

$$\text{Prob}\,(D > \text{observed value}) = Q\left(\left\{\sqrt{N_r} + 0.12 + \frac{0.11}{\sqrt{N_r}}\right\} D\right)$$

where

$$Q(t) = 2\sum_{j=1}^{\infty}(-1)^{j-1} e^{-2j^2t^2}.$$

Calculating this probability for the various proposed distributions enables us to choose the one that best fits the data for the particular situation. On carrying out this procedure we find that the pdf that fits the data best varies with angles of incidence and observation, beam-width of illumination and surface characteristics. Only for angles close to normal is the Rayleigh pdf appropriate. There is no one pdf that suits all situations. We find that different regions and situations require different pdfs. Certain pdfs are more suitable for certain situations than others.

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